- A common way to describe rotation is to use the vorticity vector ω, which is twice the rotation vector: $\omega = 2\Omega$
- In Cartesian coordinates, the vorticity is given by

$$\omega = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{k}$$

- An irrotational flow is one in which vorticity is everywhere zero.
- When applying the Bernoulli equation for irrotational flow, one can select points 1 and 2 at any locations, not just along a streamline.

Describing the Pressure Field

- The pressure field is often described using a π-group called the pressure coefficient.
- The pressure gradient near a body is related to flow separation.
  - An adverse pressure gradient is associated with flow separation.
  - A positive pressure gradient is associated with attached flow.
- The pressure field for flow over a circular cylinder is a paradigm for understanding external flows. The pressure along the front of the cylinder is high, and the pressure in the wake is low.
- When flow is rotating as a solid body, the pressure field p can be described using

$$p + \gamma z - \rho\frac{\omega^2 r^2}{2} = C$$

where ω is the rotational speed, and r is the distance from the axis of rotation to the point in the field.

Describing the Pressure Field (Summary)

Pressure variations in a flowing fluid are associated with three phenomenon:

- **Weight.** Due to the weight of a fluid, pressure increases with increasing depth (i.e., decreasing elevation). This topic is presented in Chapter 3 (Hydrostatics)
- **Acceleration.** When fluid particles are accelerating, there are usually pressure variations associated with the acceleration. In inviscid flow, the gradient of the pressure field is aligned in a direction opposite of the acceleration vector.
- **Viscous Effects.** When viscous effects are significant, there can be associated pressure changes. For example, there are pressure drops associated with flows in horizontal pipes and ducts. This topic is presented in Chapter 10 (Conduit Flow).

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3. *Turbulence*, Fluid Mechanics Films, downloaded 7/31/11 from http://web.mit.edu/hml/ncfmf.html

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5. Miller, R.W. (ed) *Flow Measurement Engineering Handbook*, New York: McGraw-Hill, 1996.

6. *Vorticity, Part 1, Part 2*, Fluid Mechanics Films, downloaded 7/31/11 from http://web.mit.edu/hml/ncfmf.html

PROBLEMS

PLUS Problem available in *WileyPLUS* at instructor's discretion.

GO Guided Online (GO) Problem, available in *WileyPLUS* at instructor's discretion.

Streamlines, Streaklines, and Pathlines (§4.1)

4.1 If somehow you could attach a light to a fluid particle and take a time exposure photo, would the image you photographed be a pathline or streakline? Explain from definition of each.

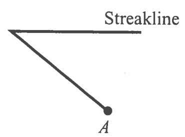
4.2 Is the pattern produced by smoke rising from a chimney on a windy day analogous to a pathline or streakline? Explain from the definition of each.

4.3 PLUS A windsock is a sock-shaped device attached to a swivel on top of a pole. Windsocks at airports are used by pilots to see instantaneous shifts in the direction of the wind. If one drew a line co-linear with a windsock's orientation at any instant, the line would be best approximate a (a) pathline, (b) streakline, or (c) streamline.

4.4 PLUS For streamlines, streaklines, and streamlines to all be co-linear, the flow must be

a. dividing
b. stagnant
c. steady
d. a tracer

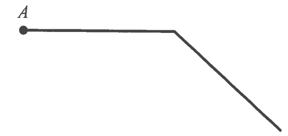
4.5 At time $t = 0$, dye was injected at point A in a flow field of a liquid. When the dye had been injected for 4 s, a pathline for a particle of dye that was emitted at the 4 s instant was started. The streakline at the end of 10 s is shown below. Assume that the speed (but not the velocity) of flow is the same throughout the 10 s period. Draw the pathline of the particle that was emitted at $t = 4$ s. Make your own assumptions for any missing information.



PROBLEM 4.5

4.6 For a given hypothetical flow, the velocity from time $t = 0$ to $t = 5$ s was $u = 2$ m/s, $v = 0$. Then, from time $t = 5$ s to $t = 10$ s, the velocity was $u = +3$ m/s, $v = -4$ m/s. A dye streak was started at a point in the flow field at time $t = 0$, and the path of a particle in the fluid was also traced from that same point starting at the same time. Draw to scale the streakline, pathline of the particle, and streamlines at time $t = 10$ s.

4.7 At time $t = 0$, a dye streak was started at point A in a flow field of liquid. The speed of the flow is constant over a 10 s period, but the flow direction is not necessarily constant. At any particular instant the velocity in the entire field of flow is the same. The streakline produced by the dye is shown above. Draw (and label) a streamline for the flow field at $t = 8$ s. Draw (and label) a pathline that one would see at $t = 10$ s for a particle of dye that was emitted from point A at $t = 2$ s.



PROBLEM 4.7

Velocity and the Velocity Field (§4.2)

4.8 PLUS A velocity field is given mathematically as $\mathbf{V} = 2\mathbf{i} + 4y\mathbf{j}$. The velocity field is:

a. 1D in x
b. 1D in y
c. 2D in x and y

The Eulerian and Lagrangian Approaches (§4.2)

4.9 PLUS There is a gasoline spill in a major river. The mayor of a large downstream city demands an estimate of how many hours it will take for the spill to get to the water supply plant intake. The emergency responders measure the speed of the leading edge of the spill, effectively focusing on one particle of fluid. Meanwhile, environmental engineers at the local university employ a computer model, which simulates the velocity field for any stage of the river, and for all locations (including steep narrow canyon sections with fast velocities, and an extremely wide reach with slow velocities). To compare these two mathematical approaches, which statement is most correct?

a. The responders have an Eulerian approach, and the engineers have a Lagrangian one
b. The responders have a Lagrangian approach, and the engineers have an Eulerian one.

Describing Flow (§4.3)

4.10 Identify five examples of an unsteady flow and explain what features classify them as an unsteady flow.

4.11 You are pouring a heavy syrup on your pancakes. As the syrup spreads over the pancake, would the thin film of syrup be a laminar or turbulent flow? Why?

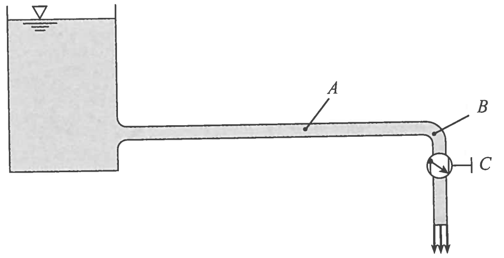
4.12 PLUS A velocity field is given by $\mathbf{V} = 10xy\mathbf{i}$. It is

a. 1-D and steady
b. 1-D and unsteady
c. 2-D and steady
d. 2-D and unsteady

4.13 Which is the most correct way to characterize turbulent flow?

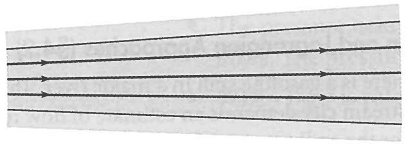
a. 1D
b. 2D
c. 3D

4.14 In the system in the figure, the valve at C is gradually opened in such a way that a constant rate of increase in discharge is produced. How would you classify the flow at B while the valve is being opened? How would you classify the flow at A?



PROBLEM 4.14

4.15 Water flows in the passage shown. If the flow rate is decreasing with time, the flow is classified as (a) steady, (b) unsteady, (c) uniform, or (d) nonuniform.



PROBLEM 4.15

4.16 If a flow pattern has converging streamlines, how would you classify the flow?

4.17 Consider flow in a straight conduit. The conduit is circular in cross section. Part of the conduit has a constant diameter, and part has a diameter that changes with distance. Then, relative to flow in that conduit, correctly match the items in column A with those in column B.

A	B
Steady flow	$\partial V_s/\partial s = 0$
Unsteady flow	$\partial V_s/\partial s \neq 0$
Uniform flow	$\partial V_s/\partial t = 0$
Nonuniform flow	$\partial V_s/\partial t \neq 0$

4.18 Classify each of the following as a one-dimensional, two-dimensional, or three-dimensional flow.

a. Water flow over the crest of a long spillway of a dam.
b. Flow in a straight horizontal pipe.
c. Flow in a constant-diameter pipeline that follows the contour of the ground in hilly country.
d. Airflow from a slit in a plate at the end of a large rectangular duct.
e. Airflow past an automobile.
f. Airflow past a house.
g. Water flow past a pipe that is laid normal to the flow across the bottom of a wide rectangular channel.

Acceleration (§4.4)

4.19 Acceleration is the rate of change of velocity with time. Is the acceleration vector always aligned with the velocity vector? Explain.

4.20 For a rotating body, is the acceleration toward the center of rotation a centripetal or centrifugal acceleration? Look up word meanings and word roots.

4.21 WILEY PLUS In a flowing fluid, acceleration means that a fluid particle is

a. changing direction
b. changing speed
c. changing both speed and direction
d. any of the above

4.22 WILEY PLUS The flow passing through a nozzle is steady. The speed of the fluid increases between the entrance and the exit of the nozzle. The acceleration halfway between the entrance and the nozzle is

a. convective
b. local
c. both

4.23 WILEY PLUS Local acceleration

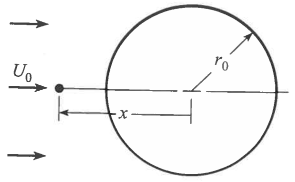
a. is close to the origin
b. is quasi nonuniform
c. occurs in unsteady flow

4.24 WILEY GO Figure 4.36 on p. 148 in §4.10 shows the flow pattern for flow past a circular cylinder. Assume that the approach velocity at A is constant (does not vary with time).

a. Is the flow past the cylinder steady or unsteady?
b. Is this a case of one-dimensional, two-dimensional, or three-dimensional flow?
c. Are there any regions of the flow where local acceleration is present? If so, show where they are and show vectors representing the local acceleration in the regions where it occurs.
d. Are there any regions of flow where convective acceleration is present? If so, show vectors representing the convective acceleration in the regions where it occurs.

4.25 WILEY PLUS The velocity along a pathline is given by $V\,(\text{m/s}) = s^2t^{1/2}$ where s is in meters and t is in seconds. The radius of curvature is 0.4 m. Evaluate the acceleration tangent and normal to the path at $s = 1.5$ m and $t = 0.5$ seconds.

4.26 Tests on a sphere are conducted in a wind tunnel at an air speed of U_0. The velocity of flow toward the sphere along the longitudinal axis is found to be $u = -U_0(1 - r_0^3/x^3)$, where r_0 is the radius of the sphere and x the distance from its center. Determine the acceleration of an air particle on the x-axis upstream of the sphere in terms of x, r_0, and U_0.



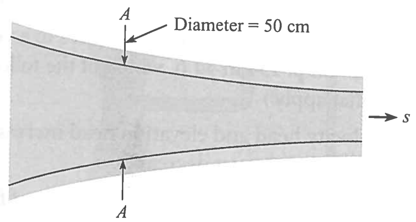
PROBLEM 4.26

4.27 WILEY GO In this flow passage the velocity is varying with time. The velocity varies with time at section A-A as

$$V = 5\text{ m/s} - 2.25\frac{t}{t_0}\text{ m/s}$$

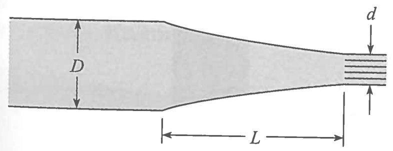
At time $t = 0.50$ s, it is known that at section A-A the velocity gradient in the s direction is $+2$ m/s per meter. Given that t_0 is 0.5 s and assuming quasi-one-dimensional flow, answer the following questions for time $t = 0.5$ s.

a. What is the local acceleration at A-A?
b. What is the convective acceleration at A-A?



PROBLEM 4.27

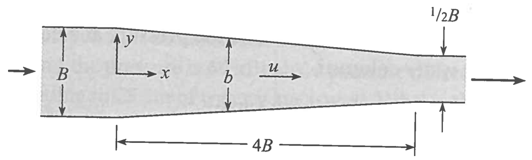
4.28 WILEY PLUS The nozzle in the figure is shaped such that the velocity of flow varies linearly from the base of the nozzle to its tip. Assuming quasi-one-dimensional flow, what is the convective acceleration midway between the base and the tip if the velocity is 0.3 m/s at the base and 1.2 m/s at the tip? Nozzle length is 46 cm.



PROBLEMS 4.28, 4.29

4.29 WILEY PLUS In Prob. 4.28 the velocity varies linearly with time throughout the nozzle. The velocity at the base is $2t$ (m/s) and at the tip is $6t$ (m/s). What is the local acceleration midway along the nozzle when $t = 2$ s?

4.30 Liquid flows through this two-dimensional slot with a velocity of $V = 2(q_0/b)(t/t_0)$, where q_0 and t_0 are reference values. What will be the local acceleration at $x = 2B$ and $y = 0$ in terms of B, t, t_0, and q_0?



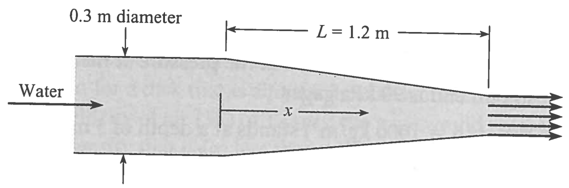
PROBLEMS 4.30, 4.31

4.31 What will be the convective acceleration for the conditions of Prob. 4.30?

4.32 WILEY PLUS The velocity of water flow in the nozzle shown is given by the following expression:

$$V = 2t/(1 - 0.5x/L)^2,$$

where V = velocity in meters per second, t = time in seconds, x = distance along the nozzle, and L = length of nozzle = 1.2 m. When $x = 0.5L$ and $t = 3$ s, what is the local acceleration along the centerline? What is the convective acceleration? Assume quasi-one-dimensional flow prevails.



PROBLEM 4.32

Euler's Equation and Pressure Variation (§4.5)

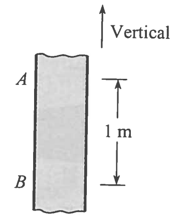
4.33 State Newton's second law of motion. What are the limitations on the use of Newton's second law? Explain.

4.34 What is the differences between a force due to weight and a force due to pressure? Explain.

4.35 A pipe slopes upward in the direction of liquid flow at an angle of 30° with the horizontal. What is the pressure gradient in the flow direction along the pipe in terms of the specific weight of the liquid if the liquid is decelerating (accelerating opposite to flow direction) at a rate of 0.4 g?

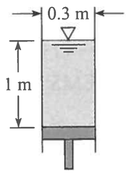
4.36 WILEY PLUS What pressure gradient is required to accelerate kerosene (S = 0.81) vertically upward in a vertical pipe at a rate of 0.5 g?

4.37 The hypothetical liquid in the tube shown in the figure has zero viscosity and a specific weight of 10 kN/m³. If $p_B - p_A$ is equal to 12 kPa, one can conclude that the liquid in the tube is being accelerated (a) upward, (b) downward, or (c) neither: acceleration = 0.



PROBLEM 4.37

4.38 If the piston and water ($\rho = 1000$ kg/m^3) are accelerated upward at a rate of $0.4g$, what will be the pressure at a depth of 0.6 m in the water column?



PROBLEMS 4.38, 4.39

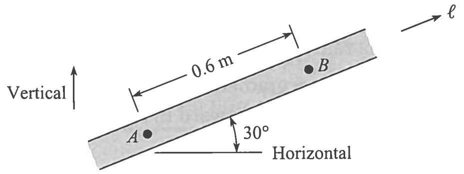
4.39 Water ($\rho = 1000$ kg/m^3) stands at a depth of 3 m in a vertical pipe that is open at the top and closed at the bottom by a piston. What upward acceleration of the piston is necessary to create a pressure of 55 kPa gage immediately above the piston?

4.40 What pressure gradient is required to accelerate water ($\rho = 1000$ kg/m^3) in a horizontal pipe at a rate of 8 m/s^2?

4.41 Water ($\rho = 1000$ kg/m^3) is accelerated from rest in a horizontal pipe that is 80 m long and 30 cm in diameter. If the acceleration rate (toward the downstream end) is 5 m/s^2, what is the pressure at the upstream end if the pressure at the downstream end is 90 kPa gage?

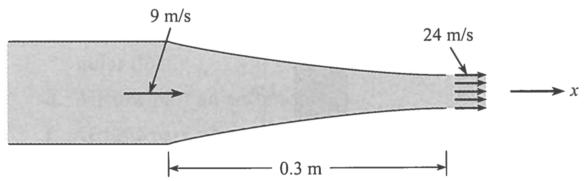
4.42 Water ($\rho = 1000$ kg/m^3) stands at a depth of 3 m in a vertical pipe that is closed at the bottom by a piston. Assuming that the vapor pressure is zero (abs), determine the maximum downward acceleration that can be given to the piston without causing the water immediately above it to vaporize.

4.43 A liquid with a specific weight of 15,700 N/m^3 is in the conduit. This is a special kind of liquid that has zero viscosity. The pressures at points A and B are 8.1 kPa and 4.8 kPa, respectively. Which one (or more) of the following conclusions can one draw with certainty? (a) The velocity is in the positive ℓ direction. (b) The velocity is in the negative ℓ direction. (c) The acceleration is in the positive ℓ direction. (d) The acceleration is in the negative ℓ direction.



PROBLEM 4.43

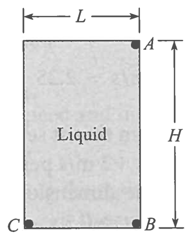
4.44 If the velocity varies linearly with distance through this water nozzle, what is the pressure gradient, dp/dx, halfway through the nozzle? ($\rho = 1000$ kg/m^3).



PROBLEM 4.44

4.45 The closed tank shown, which is full of liquid, is accelerated downward at $1.5g$ and to the right at $0.9g$. Here $L = 0.9$ m, $H = 1.2$ m, and the specific gravity of the liquid is 1.2. Determine $p_C - p_A$ and $p_B - p_A$.

4.46 The closed tank shown, which is full of liquid, is accelerated downward at $\frac{2}{3}g$ and to the right at $1g$. Here $L = 2.5$ m, $H = 3$ m, and the liquid has a specific gravity of 1.3. Determine $p_C - p_A$ and $p_B - p_A$.



PROBLEMS 4.45, 4.46

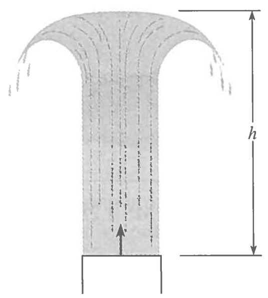
Applying the Bernoulli Equation (§4.6)

4.47 Describe in your own words how an aspirator works.

4.48 When the Bernoulli Equation applies to a venturi, such as in Fig. 4.27 on p. 134 in §4.6, which of the following are true? (Select all that apply.)

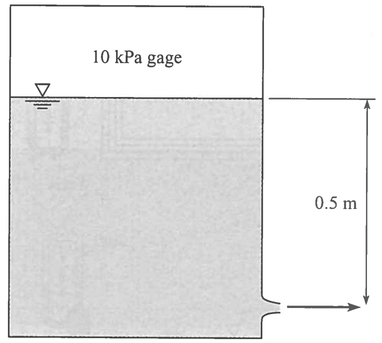
- a. If the velocity head and elevation head increase, then the pressure head must decrease.
- b. Pressure always decreases in the direction of flow along a streamline.
- c. The total head of the flowing fluid is constant along a streamline.

4.49 A water jet issues vertically from a nozzle, as shown. The water velocity as it exits the nozzle is 18 m/s. Calculate how high h the jet will rise. (*Hint:* Apply the Bernoulli equation along the centerline.)



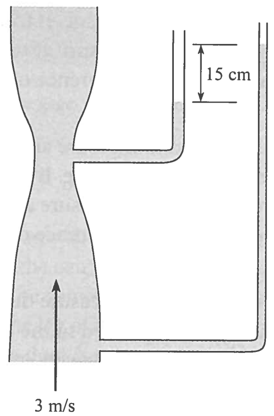
PROBLEM 4.49

4.50 A pressure of 10 kPa, gage, is applied to the surface of water in an enclosed tank. The distance from the water surface to the outlet is 0.5 m. The temperature of the water is 20°C. Find the velocity (m/s) of water at the outlet. The speed of the water surface is much less than the water speed at the outlet.



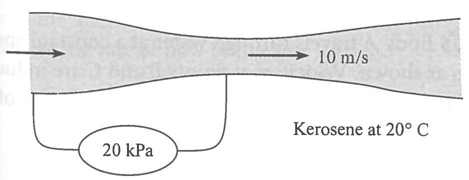
PROBLEM 4.50

4.51 Water flows through a vertical contraction (venturi) section. Piezometers are attached to the upstream pipe and minimum area section as shown. The velocity in the pipe is 3 m/s. The difference in elevation between the two water levels in the piezometers is 15 cm. The water temperature is 20°C. What is the velocity (m/s) at the minimum area?



PROBLEM 4.51

4.52 Kerosene at 20°C flows through a contraction section as shown. A pressure gage connected between the upstream pipe and throat section shows a pressure difference of 20 kPa. The gasoline velocity in the throat section is 8 m/s. What is the velocity (m/s) in the upstream pipe?



PROBLEM 4.52

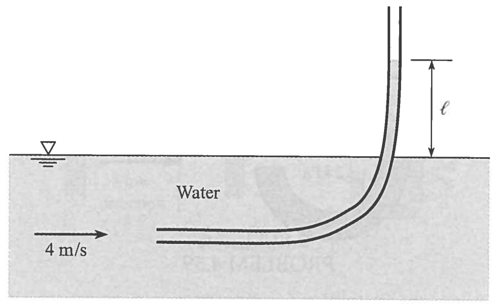
Stagnation Tubes and Pitot-Static Tubes (§4.7)

4.53 A stagnation tube placed in a river (select all that apply)

- a. can be used to determine air pressure
- b. can be used to determine fluid velocity
- c. measures kinetic pressure

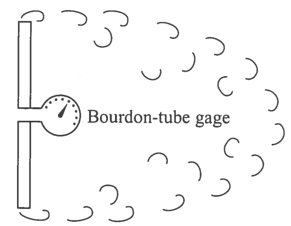
4.54 A Pitot-static tube is mounted on an airplane to measure airspeed. At an altitude of 3048 m, where the temperature is −5°C and the pressure is 69 kPa abs, a pressure difference corresponding to 25 cm of water is measured. What is the airspeed?

4.55 A glass tube is inserted into a flowing stream of water with one opening directed upstream and the other end vertical. If the water velocity is 5 m/s, how high will the water rise in the vertical leg relative to the level of the water surface of the stream?



PROBLEM 4.55

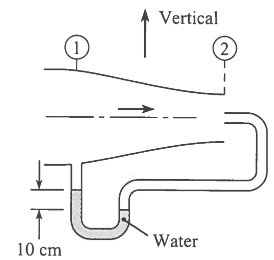
4.56 A Bourdon-tube gage is taped into the center of a disk as shown. Then for a disk that is about 0.3 m in diameter and for an approach velocity of air (V_0) of 12 m/s, the gage would read a pressure intensity that is (a) less than $\rho V_0^2/2$, (b) equal to $\rho V_0^2/2$, or (c) greater than $\rho V_0^2/2$.



PROBLEM 4.56

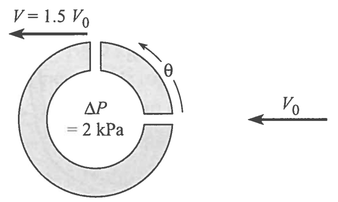
4.57 An air-water manometer is connected to a Pitot-static tube used to measure air velocity. If the manometer deflects 5 cm, what is the velocity? Assume $T = 15.5$°C and $p = 103$ kPa abs.

4.58 The flow-metering device shown consists of a stagnation probe at station 2 and a static pressure tap at station 1. The velocity at station 2 is 1.5 times that at station 1. Air with a density of 1.2 kg/m^3 flows through the duct. A water manometer is connected between the stagnation probe and the pressure tap, and a deflection of 10 cm is measured. What is the velocity at station 2?



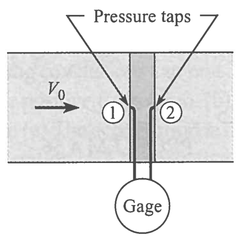
PROBLEM 4.58

4.59 The "spherical" Pitot probe shown is used to measure the flow velocity in water ($\rho = 1000\ \text{kg/m}^3$). Pressure taps are located at the forward stagnation point and at 90° from the forward stagnation point. The speed of fluid next to the surface of the sphere varies as $1.5\ V_0 \sin\theta$, where V_0 is the free-stream velocity and θ is measured from the forward stagnation point. The pressure taps are at the same level; that is, they are in the same horizontal plane. The piezometric pressure difference between the two taps is 2 kPa. What is the free-stream velocity V_0?



PROBLEM 4.59

4.60 WILEY PLUS A device used to measure the velocity of fluid in a pipe consists of a cylinder, with a diameter much smaller than the pipe diameter, mounted in the pipe with pressure taps at the forward stagnation point and at the rearward side of the cylinder. Data show that the pressure coefficient at the rearward pressure tap is −0.3. Water with a density of $1000\ \text{kg/m}^3$ flows in the pipe. A pressure gage connected by lines to the pressure taps shows a pressure difference of 500 Pa. What is the velocity in the pipe?

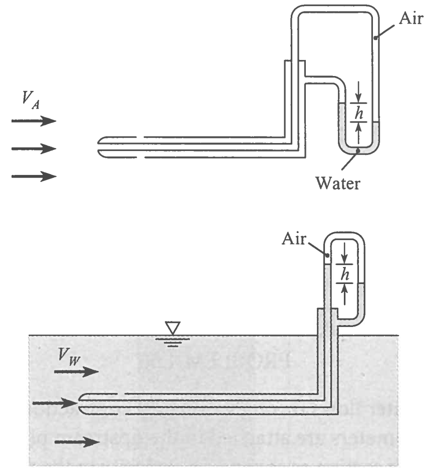


PROBLEM 4.60

4.61 Explain how you might design a spherical Pitot-static probe to provide the direction and velocity of a flowing stream. The Pitot-static probe will be mounted on a string that can be oriented in any direction.

4.62 WILEY PLUS Two Pitot-static tubes are shown. The one on the top is used to measure the velocity of air, and it is connected to an air-water manometer as shown. The one on the bottom is used to measure the velocity of water, and it too is connected to an air-water manometer as shown. If the deflection h is the same for both manometers, then one can conclude that (a) $V_A = V_w$, (b) $V_A > V_w$, or (c) $V_A < V_w$.

4.63 A Pitot-static tube is used to measure the velocity at the center of a 30 cm pipe. If kerosene at 20°C is flowing and the deflection on a mercury-kerosene manometer connected to the Pitot tube is 10 cm, what is the velocity?



PROBLEM 4.62

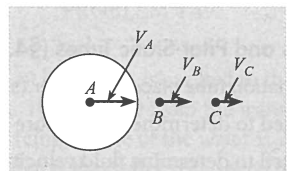
4.64 WILEY PLUS A Pitot-static tube used to measure air velocity is connected to a differential pressure gage. If the air temperature is 20°C at standard atmospheric pressure at sea level, and if the differential gage reads a pressure difference of 2 kPa, what is the air velocity?

4.65 A Pitot-static tube used to measure air velocity is connected to a differential pressure gage. If the air temperature is 15.5°C at standard atmospheric pressure at sea level, and if the differential gage reads a pressure difference of 718 Pa, what is the air velocity?

4.66 A Pitot-static tube is used to measure the gas velocity in a duct. A pressure transducer connected to the Pitot tube registers a pressure difference of 13.8 kPa. The density of the gas in the duct is $2.25\ \text{kg/m}^3$. What is the gas velocity in the duct?

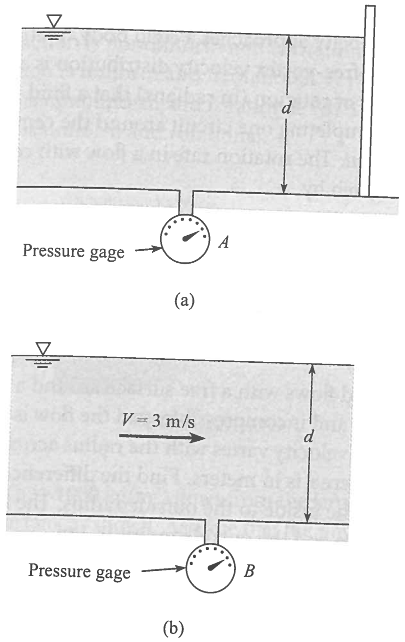
4.67 A sphere moves horizontally through still water at a speed of 3.35 m/s. A short distance directly ahead of the sphere (call it point A), the velocity, with respect to the earth, induced by the sphere is 0.3 m/s in the same direction as the motion of the sphere. If p_0 is the pressure in the undisturbed water at the same depth as the center of the sphere, then the value of the ratio p_A/p_0 will be (a) less than unity, (b) equal to unity, or (c) greater than unity.

4.68 WILEY PLUS Body A travels through water at a constant speed of 13 m/s as shown. Velocities at points B and C are induced by the moving body and are observed to have magnitudes of 5 m/s and 3 m/s, respectively. What is $p_B - p_C$?



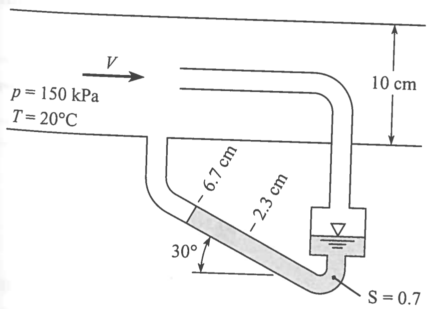
PROBLEM 4.68

4.69 Water in a flume is shown for two conditions. If the depth d is the same for each case, will gage A read greater or less than gage B? Explain.



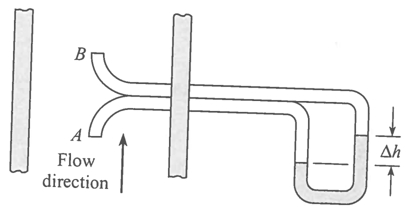
PROBLEM 4.69

4.70 WILEY GO The apparatus shown in the figure is used to measure the velocity of air at the center of a duct having a 10 cm diameter. A tube mounted at the center of the duct has a 2 mm diameter and is attached to one leg of a slant-tube manometer. A pressure tap in the wall of the duct is connected to the other end of the slant-tube manometer. The well of the slant-tube manometer is sufficiently large that the elevation of the fluid in it does not change significantly when fluid moves up the leg of the manometer. The air in the duct is at a temperature of 20°C, and the pressure is 150 kPa. The manometer liquid has a specific gravity of 0.7, and the slope of the leg is 30°. When there is no flow in the duct, the liquid surface in the manometer lies at 2.3 cm on the slanted scale. When there is flow in the duct, the liquid moves up to 6.7 cm on the slanted scale. Find the velocity of the air in the duct. Assuming a uniform velocity profile in the duct, calculate the rate of flow of the air.



PROBLEM 4.70

4.71 WILEY GO A rugged instrument used frequently for monitoring gas velocity in smokestacks consists of two open tubes oriented to the flow direction as shown and connected to a manometer. The pressure coefficient is 1.0 at A and −0.3 at B. Assume that water, at 20°C, is used in the manometer and that a 5 mm deflection is noted. The pressure and temperature of the stack gases are 101 kPa and 250°C. The gas constant of the stack gases is 200 J/kg K. Determine the velocity of the stack gases.



PROBLEM 4.71

4.72 The pressure in the wake of a bluff body is approximately equal to the pressure at the point of separation. The velocity distribution for flow over a sphere is $V = 1.5\ V_0 \sin\theta$, where V_0 is the free-stream velocity and θ is the angle measured from the forward stagnation point. The flow separates at $\theta = 120°$. If the free-stream velocity is 100 m/s and the fluid is air ($\rho = 1.2\ \text{kg/m}^3$), find the pressure coefficient in the separated region next to the sphere. Also, what is the gage pressure in this region if the free-stream pressure is atmospheric?

4.73 WILEY PLUS A Pitot-static tube is used to measure the airspeed of an airplane. The Pitot tube is connected to a pressure-sensing device calibrated to indicate the correct airspeed when the temperature is 17°C and the pressure is 101 kPa. The airplane flies at an altitude of 3000 m, where the pressure and temperature are 70 kPa and −6.3°C. The indicated airspeed is 70 m/s. What is the true airspeed?

4.74 An aircraft flying at 3048 m uses a Pitot-static tube to measure speed. The instrumentation on the aircraft provides the differential pressure as well as the local static pressure and the local temperature. The local static pressure is 67.6 kPa gage, and the air temperature is −3.9°C. The differential pressure is 3.5 kPa. Find the speed of the aircraft in km/s.

4.75 You need to measure air flow velocity. You order a commercially available Pitot-static tube, and the accompanying instructions state that the airflow velocity is given by

$$V(\text{m/s}) = 26314.7\sqrt{\frac{h_v}{d}}$$

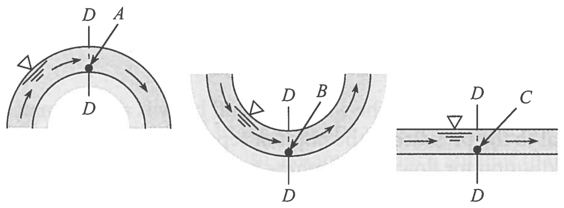
where h_v is the "velocity pressure" in meters of water and d is the density in kilogram per cubic meter. The velocity pressure is the deflection measured on a water manometer attached to the static and total pressure ports. The instructions also state the density d can be calculated using

$$d\ (\text{kg/m}^3) = 4560.2\frac{p_a}{T}$$

where P_a is the barometric pressure in meters of mercury and T is the absolute temperature in Kelvin. Before you use the Pitot tube you want to confirm that the equations are correct. Determine if they are correct.

4.76 Consider the flow of water over the surfaces shown. For each case the depth of water at section *D*-*D* is the same (0.3 m), and the mean velocity is the same and equal to 3 m/s. Which of the following statements are valid?

a. $p_C > p_B > p_A$
b. $p_B > p_C > p_A$
c. $p_A = p_B = p_C$
d. $p_B < p_C < p_A$
e. $p_A < p_B < p_C$



PROBLEM 4.76

Characterizing Rotational Motion of a Fluid (§4.8)

4.77 What is meant by rotation of a fluid particle? Use a sketch to explain.

4.78 Consider a spherical fluid particle in an inviscid fluid (no shear stresses). If pressure and gravitational forces are the only forces acting on the particle, can they cause the particle to rotate? Explain.

4.79 WILEY PLUS The vector $\mathbf{V} = 10x\mathbf{i} - 10y\mathbf{j}$ represents a two-dimensional velocity field. Is the flow irrotational?

4.80 The u and v velocity components of a flow field are given by $u = -\omega y$ and $v = \omega x$. Determine the vorticity and the rate of rotation of flow field.

4.81 The velocity components for a two-dimensional flow are

$$u = \frac{Cx}{(y^2 + x^2)} \qquad v = \frac{Cy}{(x^2 + y^2)}$$

where C is a constant. Is the flow irrotational?

4.82 WILEY PLUS A two-dimensional flow field is defined by $u = x^2 - y^2$ and $v = -2xy$. Is the flow rotational or irrotational?

4.83 Fluid flows between two parallel stationary plates. The distance between the plates is 1 cm. The velocity profile between the two plates is a parabola with a maximum velocity at the centerline of 2 cm/s. The velocity is given by

$$u = 2(1 - 4y^2)$$

where y is measured from the centerline. The cross-flow component of velocity, v, is zero. There is a reference line located 1 cm downstream. Find an expression, as a function of y, for the amount of rotation (in radian) a fluid particle will undergo when it travels a distance of 1 cm downstream.

4.84 A combination of a forced and a free vortex is represented by the velocity distribution

$$v_\theta = \frac{1}{r}[1 - \exp(-r^2)]$$

For $r \to 0$ the velocity approaches a rigid body rotation, and as r becomes large, a free-vortex velocity distribution is approached. Find the amount of rotation (in radians) that a fluid particle will experience in completing one circuit around the center as a function of r. *Hint:* The rotation rate in a flow with concentric streamlines is given by

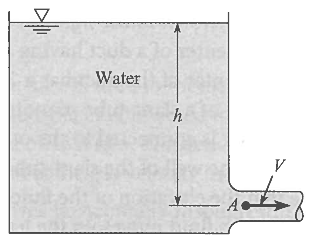
$$2\dot{\theta} = \frac{dv_\theta}{dr} + \frac{v_\theta}{r} = \frac{1}{r}\frac{d}{dr}(v_\theta r)$$

Evaluate the rotation for $r = 0.5$, 1.0, and 1.5.

The Bernoulli Equation (Irrotational Flow) (§4.9)

4.85 WILEY PLUS Liquid flows with a free surface around a bend. The liquid is inviscid and incompressible, and the flow is steady and irrotational. The velocity varies with the radius across the flow as $V = 1/r$ m/s, where r is in meters. Find the difference in depth of the liquid from the inside to the outside radius. The inside radius of the bend is 1 m and the outside radius is 3 m.

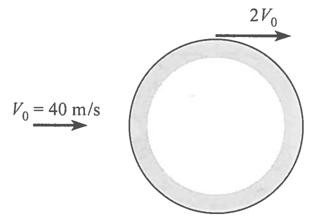
4.86 The velocity in the outlet pipe from this reservoir is 9 m/s and $h = 5.5$ m. Because of the rounded entrance to the pipe, the flow is assumed to be irrotational. Under these conditions, what is the pressure at A?



PROBLEMS 4.86, 4.87

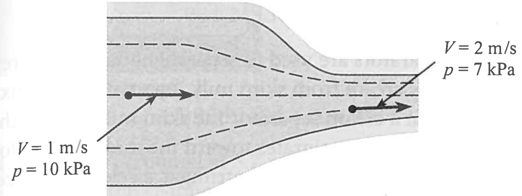
4.87 WILEY PLUS The velocity in the outlet pipe from this reservoir is 8 m/s and $h = 19$ m. Because of the rounded entrance to the pipe, the flow is assumed to be irrotational. Under these conditions, what is the pressure at A?

4.88 The maximum velocity of the flow past a circular cylinder, as shown, is twice the approach velocity. What is Δp between the point of highest pressure and the point of lowest pressure in a 40 m/s wind? Assume irrotational flow and standard atmospheric conditions.



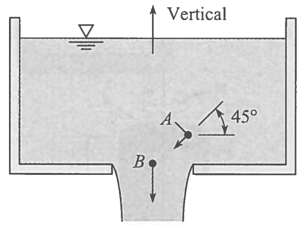
PROBLEM 4.88

4.89 The velocity and pressure are given at two points in the flow field. Assume that the two points lie in a horizontal plane and that the fluid density is uniform in the flow field and is equal to 1000 kg/m³. Assume steady flow. Then, given these data, determine which of the following statements is true. (a) The flow in the contraction is nonuniform and irrotational. (b) The flow in the contraction is uniform and irrotational. (c) The flow in the contraction is nonuniform and rotational. (d) The flow in the contraction is uniform and rotational.



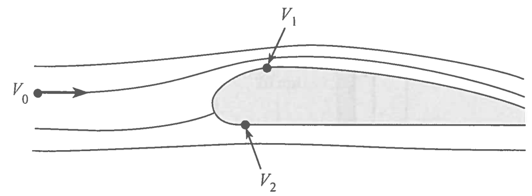
PROBLEM 4.89

4.90 Water ($\rho = 1000$ kg/m³) flows from the large orifice at the bottom of the tank as shown. Assume that the flow is irrotational. Point B is at zero elevation, and point A is at 0.3 m elevation. If $V_A = 1.2$ m/s at an angle of 45° with the horizontal and if $V_B = 3.6$ m/s vertically downward, what is the value of $p_A - p_B$?



PROBLEM 4.90

4.91 WILEY GO Ideal flow theory will yield a flow pattern past an airfoil similar to that shown. If the approach air velocity V_0 is 80 m/s, what is the pressure difference between the bottom and the top of this airfoil at points where the velocities are $V_1 = 85$ m/s and $V_2 = 75$ m/s? Assume ρ_{air} is uniform at 1.2 kg/m³.



PROBLEM 4.91

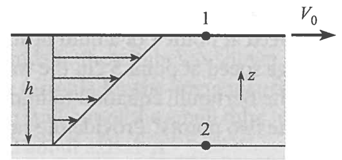
4.92 Consider the flow of water between two parallel plates in which one plate is fixed as shown. The distance between the plates is h, and the speed of the moving plate is V. A person wishes to calculate the pressure difference between the plates and applies the Bernoulli equation between points 1 and 2,

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g}$$

and concludes that

$$p_1 - p_2 = \gamma(z_2 - z_1) + \rho\frac{V_2^2}{2}$$
$$= \gamma h + \rho\frac{V^2}{2}$$

Is this correct? Provide the reason for your answer.



PROBLEM 4.92

4.93 Euler's equations for a planar (two-dimensional) flow in the xy-plane are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -g\frac{\partial h}{\partial x} \qquad x = \text{direction}$$
$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -g\frac{\partial h}{\partial y} \qquad y = \text{direction}$$

a. The slope of a streamline is given by

$$\frac{dy}{dx} = \frac{v}{u}$$

Using this relation in Euler's equation, show that

$$d\left(\frac{u^2 + v^2}{2g} + h\right) = 0$$

or

$$d\left(\frac{V^2}{2g} + h\right) = 0$$

which means that $V^2/2g + h$ is constant along a streamline.

b. For an irrotational flow,

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

Substituting this equation into Euler's equation, show that

$$\frac{\partial}{\partial x}\left(\frac{V^2}{2g} + h\right) = 0$$
$$\frac{\partial}{\partial y}\left(\frac{V^2}{2g} + h\right) = 0$$

which means that $V^2/2g + h$ is constant in all directions.

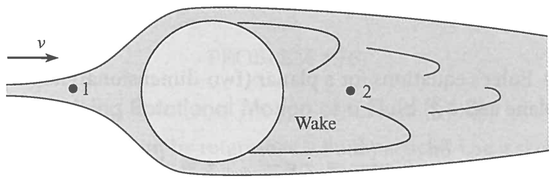
Pressure Field for a Circular Cylinder (§4.10)

4.94 A fluid is flowing around a cylinder as shown in Fig 4.37 on p. 149 in §4.10. A favorable pressure gradient can be found

- **a.** upstream of the stagnation point
- **b.** at the stagnation point
- **c.** between the stagnation point and separation point

4.95 The velocity distribution over the surface of a sphere upstream of the separation point is $u_\theta = 1.5\,U \sin\theta$, where U is the free stream velocity and θ is the angle measured from the forward stagnation point. A pressure of -6.35 cm-H_2O gage is measured at the point of separation on a sphere in a 30.5 m/s airflow with a density of 1.12 kg/m^3. The pressure far upstream of the sphere in atmospheric. Estimate the location of the stagnation point (θ). Separation occurs on the windward side of the sphere.

4.96 Knowing the speed at point 1 of a fluid upstream of a sphere and the average speed at point 5 cm the wake of in the sphere, can one use the Bernoulli equation to find the pressure difference between the two points? Provide the rationale for your decision.

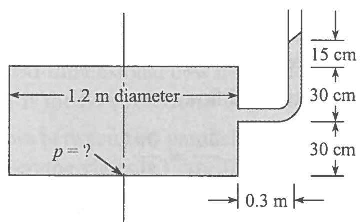


PROBLEM 4.96

Pressure Field for a Rotating Flow (§4.11)

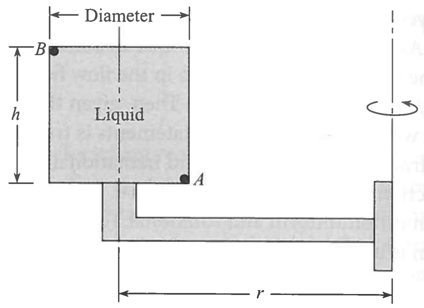
4.97 Take a spoon and rapidly stir a cup of liquid. Report on the contour of the surface. Provide an explanation for the observed shape.

4.98 This closed tank, which is 1.2 m in diameter, is filled with water ($\rho = 1000$ kg/m^3) and is spun around its vertical centroidal axis at a rate of 10 rad/s. An open piezometer is connected to the tank as shown so that it is also rotating with the tank. For these conditions, what is the pressure at the center of the bottom of the tank?



PROBLEM 4.98

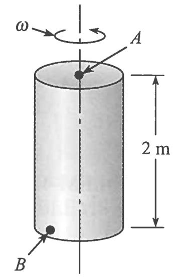
4.99 A tank of liquid (S = 0.80) that is 0.3 m in diameter and 0.3 m high (h = 0.3 m) is rigidly fixed (as shown) to a rotating arm having a 0.6 m radius. The arm rotates such that the speed at point A is 6 m/s. If the pressure at A is 1.2 kPa, what is the pressure at B?



PROBLEM 4.99

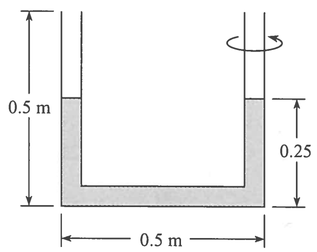
4.100 Separators are used to separate liquids of different densities, such as cream from skim milk, by rotating the mixture at high speeds. In a cream separator the skim milk goes to the outside while the cream migrates toward the middle. A factor of merit for the centrifuge is the centrifugal acceleration force (RCF), which is the radial acceleration divided by the acceleration due to gravity. A cream separator can operate at 9000 rpm (rev/min). If the bowl of the separator is 20 cm in diameter, what is the centripetal acceleration if the liquid rotates as a solid body and what is the RCF?

4.101 A closed tank of liquid (S = 1.2) is rotated about a vertical axis (see the figure), and at the same time the entire tank is accelerated upward at 4 m/s^2. If the rate of rotation is 10 rad/s, what is the difference in pressure between points A and B ($p_B - p_A$)? Point B is at the bottom of the tank at a radius of 0.5 m from the axis of rotation, and point A is at the top on the axis of rotation.



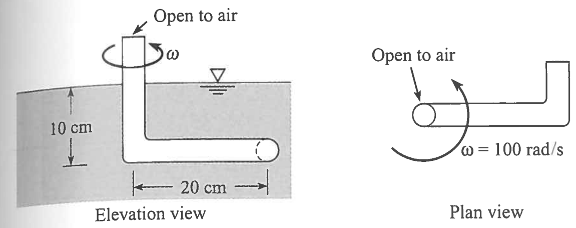
PROBLEM 4.101

4.102 A U-tube is rotated about one leg, as shown. Before being rotated the liquid in the tube fills 0.25 m of each leg. The length of the base of the U-tube is 0.5 m, and each leg is 0.5 m long. What would be the maximum rotation rate (in rad/s) to ensure that no liquid is expelled from the outer leg?



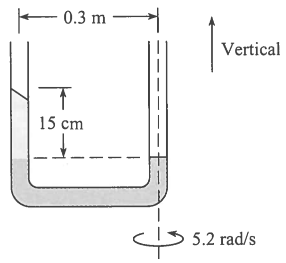
PROBLEM 4.102

4.103 An arm with a stagnation tube on the end is rotated at 100 rad/s in a horizontal plane 10 cm below a liquid surface as shown. The arm is 20 cm long, and the tube at the center of rotation extends above the liquid surface. The liquid in the tube is the same as that in the tank and has a specific weight of 10,000 N/m^3. Find the location of the liquid surface in the central tube.



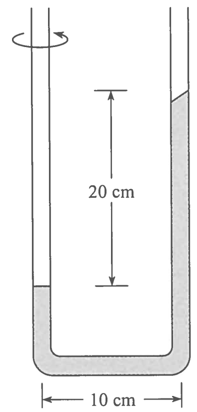
PROBLEM 4.103

4.104 A U-tube is rotated at 5.2 rad/s about one leg. The fluid at the bottom of the U-tube has a specific gravity of 3.0. The distance between the two legs of the U-tube is 0.3 m. A 15 cm height of another fluid is in the outer leg of the U-tube. Both legs are open to the atmosphere. Calculate the specific gravity of the other fluid.



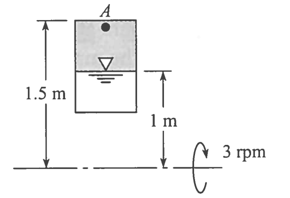
PROBLEM 4.104

4.105 A manometer is rotated around one leg, as shown. The difference in elevation between the liquid surfaces in the legs is 20 cm. The radius of the rotating arm is 10 cm. The liquid in the manometer is oil with a specific gravity of 0.8. Find the number of g's of acceleration in the leg with greatest amount of oil.



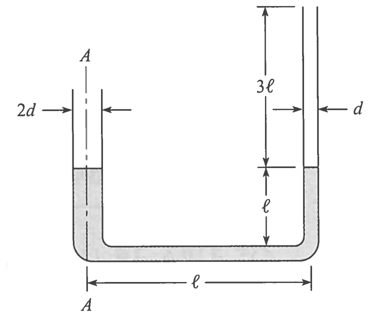
PROBLEM 4.105

4.106 A fuel tank for a rocket in space under a zero-g environment is rotated to keep the fuel in one end of the tank. The system is rotated at 3 rev/min. The end of the tank (point A) is 1.5 m from the axis of rotation, and the fuel level is 1 m from the rotation axis. The pressure in the nonliquid end of the tank is 0.1 kPa, and the density of the fuel is 800 kg/m^3. What is the pressure at the exit (point A)?



PROBLEM 4.106

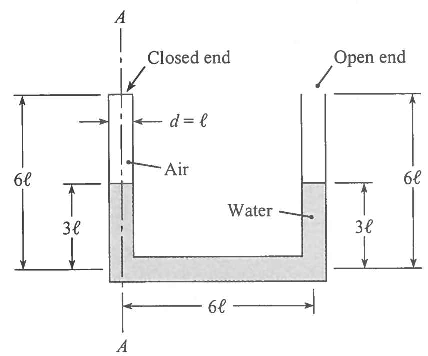
4.107 Water stands in these tubes as shown when no rotation occurs. Derive a formula for the angular speed at which water will just begin to spill out of the small tube when the entire system is rotated about axis A-A.



PROBLEM 4.107

4.108 Water ($\rho = 1000$ kg/m^3) fills a slender tube 1 cm in diameter, 40 cm long, and closed at one end. When the tube is rotated in the horizontal plane about its open end at a constant speed of 50 rad/s, what force is exerted on the closed end?

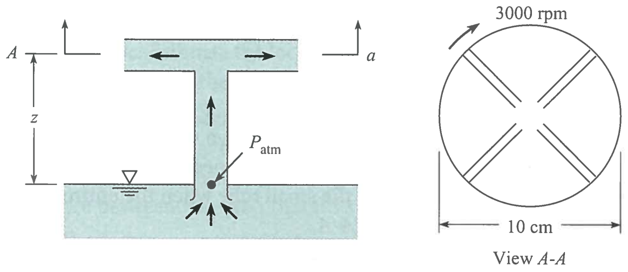
4.109 Water ($\rho = 1000$ kg/m^3) stands in the closed-end U-tube as shown when there is no rotation. If ℓ = 2 cm and if the entire system is rotated about axis A-A, at what angular speed will



PROBLEM 4.109

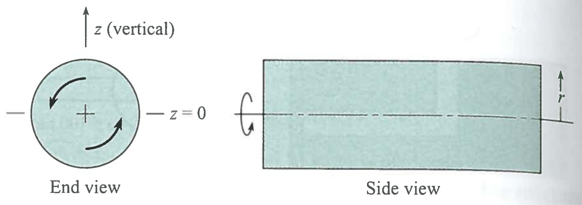
water just begin to spill out of the open tube? Assume that the temperature for the system is the same before and after rotation and that the pressure in the closed end is initially atmospheric.

4.110 WILEY PLUS A simple centrifugal pump consists of a 10 cm disk with radial ports as shown. Water is pumped from a reservoir through a central tube on the axis. The wheel spins at 3000 rev/min, and the liquid discharges to atmospheric pressure. To establish the maximum height for operation of the pump, assume that the flow rate is zero and the pressure at the pump intake is atmospheric pressure. Calculate the maximum operational height z for the pump.



PROBLEM 4.110

4.111 A closed cylindrical tank of water ($\rho = 1000$ kg/m^3) is rotated about its horizontal axis as shown. The water inside the tank rotates with the tank ($V = r\omega$). Derive an equation for dp/dz along a vertical-radial line through the center of rotation. What is dp/dz along this line for $z = -1$ m, $z = 0$, and $z = +1$ m when $\omega = 5$ rad/s? Here $z = 0$ at the axis.



PROBLEMS 4.111, 4.112

4.112 The tank shown is 1.2 m in diameter and 3.6 m long and is closed and filled with water ($\rho = 1000$ kg/m^3). It is rotated about its horizontal-centroidal axis, and the water in the tank rotates with the tank ($V = r\omega$). The maximum velocity is 7.5 m/s. What is the maximum difference in pressure in the tank? Where is the point of minimum pressure?

CONTROL VOLUME APPROACH AND CONTINUITY EQUATION 5

FIGURE 5.1

The photo shows an evacuated-tube solar collector that is being tested to measure the efficiency. This project was done by undergraduate engineering students. The team applied the control volume concept, the continuity equation, the flow rate equations as well as knowledge from thermodynamics and heat transfer. (Photo by Donald Elger.)

Chapter Road Map

This chapter describes how conservation of mass can be applied to a flowing fluid. The resulting equation is called the *continuity equation*. The continuity equation is applied to a spatial region called a control volume, which is also introduced.

Learning Objectives

STUDENTS WILL BE ABLE TO

- Define mass flow rate and volume flow rate. (§5.1)
- Apply the flow rate equations. Describe how the flow rate equations are derived. (§5.1)
- Define and calculate the mean velocity. (§5.1)
- Describe the types of systems that engineers use for analysis. List the key differences between a CV and a closed system. (§5.2)
- Describe the purpose, application, and derivation of the Reynolds transport theorem. (§5.2)
- Describe and apply the continuity equation. Describe how the equation is derived. (§5.3, §5.4)
- Explain what cavitation means, describe why it is important, and list guidelines for designing to avoid cavitation. (§5.5)